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The $N_\gamma(\frac{3}{2}^-)$ (1520) resonance in $\pi N \to \pi\Delta$

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Abstract. We present theoretical and experimental evidence for a large d-wave $\pi\Delta$ decay for the $N_\gamma(\frac{3}{2}^-)$ (1520) resonance although the s-wave decay is kinematically preferred by a factor of about 19. The theoretical method here is a simple low energy bootstrap model, with resonance saturated dispersion relations (consistent with the Harari–Freund hypothesis). It agrees well with the dip structure required by ρ–baryon duality and also with experiment.

In a recent paper (York 1972) we presented an approximate resonance saturated fixed-t dispersion relation model for the reaction $\pi N \to \pi\Delta$, and showed how it could be used as a bootstrap scheme for predicting resonances. In particular we found a strong triple bootstrap involving the N, Δ and $N'_\alpha(\frac{1}{2}^+)$(1470) states. Since the $N_\gamma(\frac{3}{2}^-)$(1520) resonance was experimentally known to be strong in $\pi N \to \pi\Delta$ we examined the DS13 partial wave but could find no evidence sufficient to predict this resonance. The DD13 wave was neglected since the extra q^2 factor was sufficiently small that we did not anticipate that any d-wave contribution would materially affect anything. We shall show here, however, that the DD13 wave provides an excellent opportunity for predicting the $N_\gamma(\frac{3}{2}^-)$ resonance in this bootstrap scheme, and thus providing excellent agreement between our model and experiment.

We can summarize the method of predicting resonances in the following way†.

We calculate various contributions for the left-hand cut (LHC) singularities to the s-channel physical region of the particular partial wave. If this LHC term is sufficiently large that it exceeds the unitarity limit on the real part above some energy s_1 then we argue that further contributions to $a(s)$ with a significant imaginary part below s_1 are necessary, and should be sufficient to give a real part contribution which cancels $a_{\mathrm{LHC}}(s)$ above s_1, thus restoring the unitarity limit. If s_1 is near threshold and we believe $a_{\mathrm{LHC}}(s)$ to be a good approximation to u-channel exchanges then we predict an s-channel resonance.

For the DD13 wave in $\pi N \to \pi\Delta$, u-channel Δ exchange‡ dominates the left-hand cut and is strongly negative, exceeding the unitarity limit§ above $s_r \simeq 3{\cdot}5\ \mathrm{GeV}^2$ (see figure 1). If we were to include experimental elastic phase shift information on the D13 wave elasticity, we would have an even smaller unitarity limit by a factor $(1-\eta^2)^{1/2}$ and the point s_r brought down to $3{\cdot}15\ \mathrm{GeV}^2$. Thus we can predict the $N_\gamma(\frac{3}{2}^-)$ resonance with

† For a fuller description consult York (1972) and Lyth and York (1972).

‡ We use $\gamma = 13{\cdot}0\ \mathrm{GeV}^{-1}$ predicted by York (1972) as the input Δ coupling.

§ We use reduced partial waves so that in this case the unitarity limit is $\pm 1/2q_{\pi N}^{5/2}q_{\pi\Delta}^{5/2}$.

$M_r^2 \lesssim 3{\cdot}15\,\text{GeV}^2$. In order to restore the unitarity limit we require its reduced coupling to lie in the range (if $M_r \sim 1520\,\text{MeV}$)

$$-16 \lesssim \gamma \lesssim -4{\cdot}5\,\text{GeV}^{-3}. \tag{1}$$

We show in figure 1 the result of including the resonance with $\gamma = -10\,\text{GeV}^{-3}$, $M_r = 1520\,\text{MeV}$ and $\Gamma = 120$.

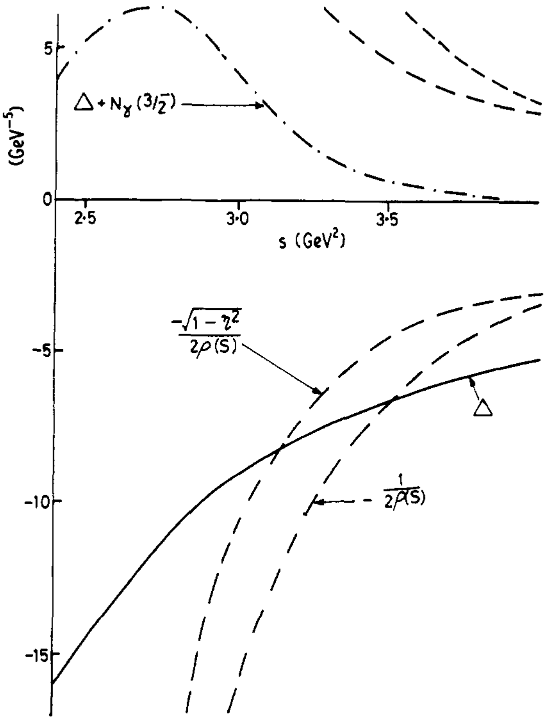

Figure 1. Contributions to the real part of $a(s)$ for *DD*13 partial wave. The broken curves represent the unitarity limits with and without the factor $(1-\eta^2)^{1/2}$.

In terms of branching ratios, (1) gives

$$-0{\cdot}23 \lesssim (\chi_{\pi N}\chi_{\pi\Delta}(D))^{1/2} \lesssim -0{\cdot}07. \tag{2}$$

We thus conclude that d-wave decay of the $N_\gamma(\frac{3}{2}^-)$ state is important and may even be comparable in experimental magnitude to the s-wave.

This result ties up with the duality prediction of Gell *et al* (1971) who show that in order to reproduce the nonsense wrong signature zero of ρ-exchange from baryon resonances, each resonance must preferentially produce the final Δ in a helicity $\pm\frac{3}{2}$ state. This can be interpreted as a constraint on the relative decay rates into the two possible orbital states (except for spin $\frac{1}{2}$ resonances):

$$\left(\frac{\chi_{l+2}}{\chi_l}\right)^{1/2} \simeq \begin{cases} \left(\dfrac{3(l+1)}{l+3}\right)^{1/2} & \text{if } J = l+\frac{3}{2} \\ \left(\dfrac{l}{3(l+2)}\right)^{1/2} & \text{if } J = l+\frac{1}{2}. \end{cases} \tag{3}$$

Thus resonances with $J = l+\frac{3}{2}$ will generally prefer higher waves and those with $J = l+\frac{1}{2}$, lower waves.

This has already been verified with the following resonances†: $N_\gamma(\frac{5}{2}^-)$, $\Delta_\delta(\frac{7}{2}^+)$, $\Delta_\alpha(\frac{5}{2}^+)$ (see Gell *et al* 1971, Mehtani *et al* 1972). For the $N_\gamma(\frac{3}{2}^-)$ the prediction (3) is

$$\left(\frac{\chi_D}{\chi_S}\right)^{1/2} \simeq 1.0 \tag{4}$$

which is in approximate agreement with a recent phase shift analysis (Herndon *et al* 1972) which gives DS13 and DD13 waves about the same at $W = 1520$ MeV.

For our analysis however, it is the reduced couplings that are important and in this case the extra $1/q^2_{\pi\Delta}$ factor makes the dynamical d-wave preference quite clear:

$$\frac{\gamma_D}{\gamma_S} \simeq 19\ \text{GeV}^{-2}.$$

We have attempted to fit data on $\pi^- p \to \pi^+\Delta^-$‡ between 1490 and 1533 MeV (CM energy) using the approximate resonance saturated fixed-t dispersion relation model (York 1972) which is really a sum of second sheet poles in the regularized t-channel helicity amplitudes. We have used the three states predicted earlier ($N, \Delta, N'_\alpha(\frac{1}{2}^+)$) together with $N_\gamma(\frac{3}{2}^-)$ in both s- and d-wave $\pi\Delta$ states.

Our nine parameters are the five reduced couplings and the masses and widths of the $N'_\alpha(\frac{1}{2}^+)$ and $N_\gamma(\frac{3}{2}^-)$. The N coupling was constrained to lie near 26·5 GeV^{-1} (given by the $\Delta \to \pi N$ coupling $g^{*2}/4\pi = 0.30 M_\pi^{-2}$ and $g^2/4\pi = 14.6$).

In table 1 we show the range of the parameters resulting from the fit. In figure 2 we show the fit for the particular solution of table 2. The χ^2 is 161 for 82 data points. It can be seen from figure 2 that the poor χ^2 is due almost entirely to inconsistency in the data. (Compare for example the histograms for $W = 1530$ and $W = 1533$ which should be the same§.)

The fitted masses are lower than usual and this is probably due to the fact that they correspond to the real part of the pole position, rather than the zero of the real part of the

Table 1. Variation of parameters for good fits ($\chi^2 = 160$–165)

		γ	M	Γ
N		$21 \to 25$[a]	939[b]	—
Δ		$10 \to 14$	1232[b]	120[b]
$N'_\alpha(\frac{1}{2}^+)$		$5 \to 9$	$1400 \to 1436$	$188 \to 233$
$N_\gamma(\frac{3}{2}^-)$	s-wave	$-0.4 \to -0.7$	$1485 \to 1499$	$91 \to 113$
	d-wave	$-8 \to -11$		

a Constrained to lie near 26·5.
b Fixed.

† The agreement, at the moment, however, only extends to which wave is preferred. The quantitative agreement is not yet very good.

‡ The histograms for $W = 1490$, 1530 come from the Saclay experiment (see De Beer *et al* 1969), and I would like to thank G Smadja for communicating these. The histograms for $W = 1501$, 1533 come from the SLAC–LRL collaboration (see Brody *et al* 1971), as presented in Brody *et al* 1968.

§ The Saclay histogram has a narrower mass cut in the Δ selection, but still contains more events and is therefore probably more reliable.

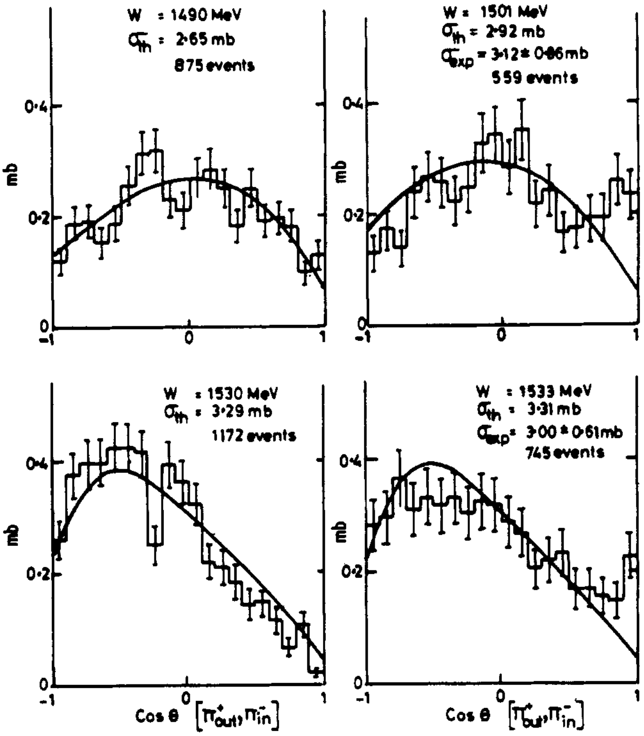

Figure 2. Fit to angular distribution histograms for $\pi^- p \rightarrow \pi^+ \Delta^-$.

Table 2. Parameters used in fit shown in figure 2

		γ	M	Γ
N		23·0	939	—
Δ		11·9	1232	120
$N'_\alpha(\frac{1}{2}^+)$		7·2	1417	209
$N_\gamma(\frac{3}{2}^-)$	s-wave	−0·5	1490	101
	d-wave	−9·5		

amplitude. Adding the other contributions (mostly LHC) shifts the zero to a higher energy more compatible with the traditional resonant energies.

In table 3 we compare the theoretical predictions to the fitted values. The agreement is remarkably good and is strong confirmation of the resonance saturation hypothesis which follows from two-component duality (Harari 1968, Freurd 1968).

The striking result is the ratio of $N_\gamma(\frac{3}{2}^-)$ couplings, in excellent agreement with the duality prediction and the bootstrap argument presented here. The large d-wave contribution was vital to obtaining a good fit, and it can be seen from figure 2 that the data do indeed have the backward dip required by duality† ($|t|_{max} \simeq 0{\cdot}45\ \mathrm{GeV}^2$ at $W = 1530$ MeV) and reflecting the strong d-wave contribution.

† This dip prediction is, of course, only for $I_t = 1$ exchanges, whereas the data are pure $I_t = 2$. However, since the theoretical amplitudes (and the data) are nearly all $I_s = \frac{1}{2}$ in this region, the dip should also be present for $I_t = 2$.

Table 3. Comparison of theory with fit

		Theory	Fit
N		$\gamma = 26{\cdot}5$	23 ± 2
Δ		$\gamma \gtrsim 13\dagger$	12 ± 2
$N'_\alpha(\frac{1}{2}^+)$		$\gamma \gtrsim 6\dagger$	7 ± 2
$N_\gamma(\frac{3}{2}^-)$	s-wave	small	$-0{\cdot}5 \pm 0{\cdot}1$
	d-wave	$-16 \lesssim \gamma \lesssim -4{\cdot}5\dagger$	$-9{\cdot}5 \pm 1{\cdot}0$
γ_D/γ_S		19	20 ± 5

† Dependent on $\gamma_N = 26{\cdot}5$. Can be reduced in approximate proportion.

The ratio of d-wave to s-wave couplings for the $N_\gamma(\frac{3}{2}^-)$ state is also of significance for higher symmetries. Our result favours the opposite sign to that of strict $SU(6)_w \times O(2)_{L_z}$ (Petersen and Rosner 1972, Faiman and Plane 1972) similar to that which has been found for some meson decays (Colglazier and Rosner 1971).

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